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«Identity, Fuzziness And Noncontradiction»

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IDENTITY, FUZZINESS AND NONCONTRADICTION
by Lorenzo Peña

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§0.— Introduction

Old dialectical arguments seem to show that self-identity is false. Those arguments find themselves reinforced by contemporary fuzzy approaches, which, owing to inference rules apparently sound, yield the same conclusion. But, is that conclusion unacceptable?

In this paper I argue that fuzziness, instead of enjoining the rejection of the principles of excluded middle and noncontradiction, entails those principles' truth, even though it also entails negations of some instances thereof. Thus, classical logic needs to be weakened, by distinguishing simple negation from strong negation or overnegation. Disjunctive syllogism is only valid for the latter. So, there may be mutually contradictory truths — including both self-identity and self-distinction.

That line of argument leads to rejecting the maximality rule, according to which only altogether true sentences are true, and embracing instead the rule of endorsement, which means that whatever is more or less true is true.

Backing those conclusions up is made in light of a discussion about the problem of the purported ontological neutrality of logic, which is proved to be a delusion. Every logical principle is ontologically committed, at least in a negative way, which is no less real than the positive one. What makes fuzzy logic worth working on is not its greater ontological neutrality but whatever arguments can be put forward in support of a fuzzy ontology.

Throughout this paper symbolic notation is sometimes used. I resort to Churchian conventions concerning dots. In restoring omitted parentheses, a dot written immediately after an occurrence of any two-place functor indicates a left parenthesis with its mate as far to the right as possible. Remaining ambiguities are resolved by associating leftwards. Double quotes, used like corners, are sometimes elided.

§1.— Is everything distinct from itself?

There are several arguments which, apparently at least, show that everything is distinct from itself. I go on to muster them.

The first one is Wittgenstein's puzzle about identity. Nothing is the same as anything else, but can you say that a thing bears to itself the relation of identity? Kripke replies ([14]: 350, n. 50) that many relations are reflexive, and identity is just one of them. But that reply is not sufficient, for, what turns out to be queried is just the very possibility of non-irreflexive relations. For, a relation is something that relates, and relating is tying, or fastening together or linking, one thing to another thing; so, some kind of otherness is bound to exist between the things related.

A second argument for self-distinction of any object is based on the by now usual way of regarding relations as sets of ordered pairs. Fatherhood relation, e.g., would be a set one of whose members is the set {{Abraham}, {Abraham, Isaac}}, the other members being of the same ilk. But what about a non-irreflexive relation, a relation which may hold between some thing and itself? Suppose that Silas fondles himself; then one member of the fondling-relation will be ({Silas}, {Silas, Silas}}, i.e. {{Silas}}. Now, in order for {{Silas}} to be an ordered *pair*, some kind of distinction or other must exist between Silas and himself.

The argument above doesn't apply to a combinatorial categoryless theory of relations, according to which facts (= states of affairs or propositions) are properties or sets, and conversely. Such a theory identifies Troilus' loving Cressida both with Cressida's exemplifying Troilus' love (Troilus' love being nothing else but Troilus's being a lover) and with Troilus' exemplifying Cressida's being loved. This combinatory theory of relations has been put forward by the present writer in [32] and something like it has been proposed by F. Fitch ([8], [9]). However, another argument can be applied to the combinatorial theory of relations, to wit: Macbeth's killing Duncan is something both of Macbeth and of Duncan, but in different ways: it's of Macbeth as killer and of Duncan as being-killed. The unenlightening 'as' can be elucidated here by saying that killing is a property of Macbeth, while Macbeth's killing is a property of Duncan. But what about Othello's killing himself? Then, the killing relation shall also apply to Othello and Othello in two different ways. Now, we can take it that whenever one same entity constitutes (or pertains to) «two» things in two different ways, those «two» things ought to be distinct from one another — there must be some kind of otherness between them.

A third argument for self-distinction rests upon three premises, two of them usually regarded as tautologies, the third one being a highly controversial claim that I for one take to be true. I'll avail myself of the following symbols: '$\leftrightarrow$' which stands for equivalence («$p\leftrightarrow q$» being read: «the fact that p is equivalent to the fact that q» or «the fact that p and the fact that q are implied by one another»); 'N' which is simple negation: 'It's not the case that'; '.' which is conjunction; «$p\rightarrow q$» abbreviates «$p\wedge q\leftrightarrow p$» ('$\rightarrow$' being implication). The following premises are understood as applying to any formulas «p» and «q»:

(1st. Premise) $p\leftrightarrow p\leftrightarrow .q\leftrightarrow q$

(2d. Premise) $p\rightarrow q\rightarrow N(p\wedge Nq)$

(3d. Premise) $N½\leftrightarrow½$

Let the symbol '½' stand for any sentence which happens to be as true as false. Here lies the stumbling block in my argument. I well know that champions of classical logic will balk at admitting that there may be such a sentence. However, the fruitfulness of fuzzy approaches has been sufficiently proved in many sciences, and, accordingly, accepting that some sentences may be as true as false shouldn't be startling. Surely there is some point at which a man is as bald as not-bald; some amount of sand which, to the same extent, both is and is not a heap; some degree of parsimony which renders both, and to the same extent, stingy and not-stingy; some link in evolution at which some animals both are and, in the same degree, fail to be mammals; some battles which are as won as lost by an army; dialects which, in the same degree, both belong and don't belong to a certain language; amphibians which are as terrestrial as aquatic; pulls which can be described half truly and half falsely as tugs; some degree of unhealthiness or indisposedness at which people are as ill as not-ill; some lack of refinement such that those who have it can be said to be as boorish as not boorish; some point at which humility or obedience is, and to the same extent is not, truckling.

Anyway, if we accept the abduction rule, the 3d premise yields (4):

(4) $½\wedge N½$

Whence, in virtue of (2d. Prem.), contraposition and *modus ponens*, we get:

(5) $N(½\rightarrow½)$

which in turn yields (6) — since, for any «p», «p» and «p∧p» are equivalent:

(6) $N(½\leftrightarrow½)$

Now, in virtue of (1st. Prem.), (6) entails (7), for any «p»:

(7) $N(p\leftrightarrow p)$

Regardless of which definition of identity we happen to choose, we obviously want to define identity by way of equivalence. Hence, by standard rules we now conclude, for every x:

(8) $N(x=x)$

The conclusion could be strengthened. Since (3d. Prem.) entails that '½∧N½' is at least half true, we have — in virtue of (2d. Prem.) and contraposition — that '½→½' — which is equivalent to '½↔½' — is at least half false; hence — in virtue of (1st. Prem.), every sentence of the form «p↔p» is at least half false. Consequently, 'x=x' is at least half false, too.

We can now conclude that every self-equivalence sentence is at least half true, too — and hence exactly as true as false — if we add a fourth premise, viz. (9):

(9) $p\wedge Np\rightarrow.p\leftrightarrow p$

The argument may be given another, perhaps tighter, form. From the rule δp[x], δNp[x/y] ⊦ δN(x=y) — where 'δ' stands for any extent-functor, meaning 'It's, (at least) to … degree, true that' — and from the hypothesis of '½' being a sentence as true as false, we draw, in virtue of absorption (i.e. that, for any «p» and «q», «p∨q∧p↔p», where '∨' is disjunction) the conclusion: δN(x=x), for any x.

We thus seem to be compelled to conclude, from the highly plausible hypothesis that some propositions are as true as false, that every entity is both, and in the same degree, identical to itself and distinct from itself.

Interesting corollaries of (7) above are Aristotle's and Boethius' principles, viz: «p→q→N(p→Nq)» and «N(p→Np)». In fact, either of those principles could replace (2d. Prem.) in our previous reasoning.

The upshot of the foregoing discussion is that we have a good argument in support of the principle of distinction (P.D. henceforward), viz.: *everything is rather distinct from itself* (where 'rather' abbreviates 'it's at least half true that'). The argument relies on a the fuzzy principle (3d. Prem.), together with logical tautologies like (1st. Prem.), (2d. Prem.), idempotence of conjunction, abduction (p→Np→Np and Np→p→p), contraposition (p→q→. Nq→Np), substitutivity of equivalents (p↔q→.(…p---)↔(…q---)), as well as *modus ponens*. We nowise need to have all classical tautologies. Nor do we want to, since we are not prepared to draw «q» (any «q») from (3d. Prem.), according to Pseudo-Scotus' law, which is a classical tautology.

I suppose the most likely objection against this reasoning will be that whosoever accepts fuzziness, and so is liable to admit (3d. Prem.), will reject abduction. But, why should he? The reply would then be that fuzziness together with abduction entails contradictions. Well, since that is so, my own answer is that he who accepts fuzziness should accept contradictions, too. But more on this below.

A fourth argument for self-distinction is much less general, but can be generalized. Let's take an entity which happens to go by two different names, like the one named both 'St. Paul' and 'Saul of Tarsus.' It may happen that someone knows of St. Paul that he is St. Paul, while not knowing of Saul of Tarsus that he is St. Paul. Many solutions have been contrived: distinctions of reason (which solution can be reshaped and made respectable through a semantics befitting substitutional treatment of quantifiers, see [15]); giving up full-blown indiscernibility of identicals; construing belief-contexts as opaque (as including disguised quotations), and so on. Although my own position would be sympathetic to regarding as partly quotational *some* belief-contexts, I think it preferable not to resort to such a construal when we can do without it. As for all other proposals, they are fraught with disadvantages which are too heavy. Take, e.g., the one of waiving indiscernibility of identicals. I cannot imagine what it would be for one same entity (i.e. for «two» things which are one and the same entity) to be discernible from itself, that is to say: to exemplify *and* to lack, both in a high degree — in a degree, that is, of more than fifty percent — some property or other. (If waiving indiscernibility is construed as regarding «contingent identity» as a looser relation than identity proper, then that solution in addition to an obnoxious multiplication of entities leads to a loss of any general identity-criterion, since dubbing an entity in a new way would suffice to disclose (or even to create?) an irreducible entity, not-identical with what one aimed at naming.)

An alternative solution would lie in relativising identity: Saul of Tarsus, although the same man as St. Paul, wouldn't be the same apostle as him. Although I also find unattractive such a solution, I do not propose to discuss it here.

Unless we take up one of those proposals, I see no alternative left but stating that Saul of Tarsus both is the same and is not the same as St. Paul, and, consequently, whosoever does not think that Saul of Tarsus is St. Paul, while thinking that St. Paul is St. Paul, both thinks and does not think that St. Paul is St. Paul (in virtue of indiscernibility of identicals).

Of course anything whatever could get two different names. Thus our conclusion could be generalized.

Before proceeding on, I want to foil a probable objection, namely: since self-distinction is an unpalatable result, something or other must be wrong in the premises of every argument backing up that conclusion. This objection is hard to cope with rigorously. My answer, though, is that unpalatableness is relative to a viewpoint. If you cleave to rejecting self-distinction, you're lead to also reject something or other in all and every argument yielding this conclusion. Now, even if some premises may sound dubious, it still seems to be that junking at least a premise in every one of those four arguments is — to say the least — far less enticing than accepting the conclusion. Once we see the matter this way, we are liable to get acquainted with self-distinction and sympathetic towards the dialectical tradition which has held that claim. Then, the kind of Moore-wise objection I'm now parrying becomes less and less convincing. (Such is the case as regards any perplexing conclusions gotten at through apparently cogent arguments. I'm not saying that we are «free» to choose either *modus ponens* or *modus tollens*. We don't feel «free»: some option or other may be causally determined in each case. Now, no option is of itself self-evidently and unproblematically correct.)

§2.— Why fuzziness requires the truth of excluded middle

Having reached the conclusion that everything is distinct from itself, I must admit that my most cogent argument was the third one, relying on fuzziness. This section's goal is to discuss the relationship between fuzziness and the principles of noncontradiction and excluded middle.

It has been contended — mistakenly, I think — that fuzziness or vagueness means either failure of the principle of excluded middle or lack of truth-value. I go on to set out my reasons for rejecting those claims.

Let's start with the contention that fuzziness means truth-valuelessness (see, e.g., [37]). That opinion either rests on assuming two-valued-ness (and so regarding as truth-value-less whatever is such that, should it have any truth-value, that truth-value would be a nonclassical one), or else relies on some independent ground. But which ground? I suppose that what prompts its followers to maintain it is thinking that whenever some fuzzy situation arises, the problem is that some concept is not well-defined, so that we cannot say that it applies to the situation, but we cannot say either that it doesn't apply. Thus, as regards such situations we can say nothing, for our terms are ill-defined.

I think that is untrue. Ill-definedness has nothing to do with fuzziness. Should a man fly, probably we'd be nonplussed, without knowing what to say: is «he» a flying man? Or is «it» a manlike flying creature? Problems of that ilk, whether spurious or not, are entirely different from fuzziness-problems. When a fuzzy situation arises, we do very often know what to say: we say that some entity involved in the situation is such that it neither satisfies nor fails to satisfy some given predicate. «Fronterizo» dialect in Uruguay neither is nor fails to be Spanish.

True, some fuzzy situations are perplexing. Some people don't know whether they can assert, or deny, or both, or neither, whether some land is barren, or desert. Nevertheless, most perplexities of that sort originate with the prejudice that only what is wholly true is true (such is the principle of maximality, which I intend to refute). Be that as it may, most fuzzy situations do not usually bring about such perplexities. We commonly say that many people are neither happy nor unhappy (not that their being happy is ill-defined, or truth-value-less, but that they lack both happiness and its complement).

Let's now address ourselves to the question of whether fuzziness means failure of the principle of excluded middle. This principle says that, for every «p», «p∨Np» (in words: 'Either p or it's not the case that p») is true. Those who contend that fuzziness entails failure of excluded middle seem to think that a fuzzy situation, say Guinea's being a French-speaking country, is such that we can neither state its truth nor state its falsity. Since both its truth and its falsity fail to hold, the disjunction of them fails to hold, too.

My reply is that what happens in such a case is not that we can neither state nor deny that Guinea is a French-speaking country, but that we can and do say that Guinea neither is nor fails to be a French-speaking country, i.e. that it's not the case that Guinea is a French-speaking country and it's not the case either that Guinea is not a French-speaking country; we thus have a statement of the form 'neither p nor not-p», which, by simplification, entails «not-p», which in turn does by addition entail «p or not-p». So, instead of fuzzy situations compelling us to say nothing about them, or to say nothing about something's exemplifying some property, they allow us both to say that the thing under consideration fails to exemplify the property and that still it also fails to fail to exemplify it. (Examples can be multiplied. What we say, for instance, is that ichthyostegas neither are nor fail to be fishes; that the Sahel neither is nor fails to be a desert land; that the 14th century's second half neither belongs nor fails to belong to the Renaissance period, and so on.)

The upshot of the foregoing considerations is that not only is it false that fuzziness entails failure of excluded middle, but, what is more, any fuzzy situation entails that excluded-middle applies thereto. Nonetheless, fuzzy situations do also entail negations of instances of excluded-middle. Let's take as an example eleven-carat gold being gold. We usually say that eleven-carat gold neither is gold nor fails to be gold. Now, by De Morgan's Law, that statement is (and is commonly felt to be) equivalent to: 'It's not the case that: either eleven-carat gold is gold or else eleven-carat gold is not gold'. Let's abbreviate as 'r' the sentence 'Eleven-carat gold is gold'. Then what we commonly say is: «$Nr\wedge NNr$», which is a contradiction or antinomy and, in virtue of De Morgan, is equivalent to «$N(r\vee Nr)$», a negation of an instance of excluded middle. Of course, once a contradiction of the form «$p\wedge Np$», which we can abbreviate as «Sp», is gotten at, the principle of noncontradiction along with adjunction allow us to draw infinitely many contradictions therefrom. For we'll have: NSr (instance of noncontradiction); Sr (hypothesis); SSr (by adjunction); NSSr (instance of noncontradiction); SSSr, and so on and so forth.

Thus, fuzzy situations are shown to be such situations as make some contradictions true. But neither the principle of noncontradiction nor the principle of excluded middle fail when a fuzzy situation (hence, a contradiction) is the case. Coping with fuzziness does not compel us to drop those ontological principles, but to add something thereto: to add contradictions. Hence, Peirce was right when he wrote to William James, in 1909:

> I have long felt that it is a serious defect in the existing logic that it takes no heed of the *limit* between two realms. I do not say that the Principle of Excluded Middle is downright *false;* hut I *do* say that in every field of thought whatsoever there is an intermediate ground between *positive assertion* and *positive negation* which is just as Real as they. Mathematicians always recognize this; and seek for that limit as the presumable lair of powerful concepts; while metaphysicians and old-fashioned logicians — the sheep & goat separators — never recognize this. The recognition does not involve any denial of existing logic, but it involves a great addition to it. ([10]: 81).

Acknowledging fuzziness is no ground for waiving the principles of noncontradiction and excluded middle. It offers, though, a strong motivation for distinguishing those right principles from two fallacious ones, resulting therefrom by prefixing to them the functor 'wholly'. The results will be called: the principle of absolute noncontradiction (P.A.N.) and the principle of absolutely excluded middle (P.A.E.M.). Both are wrong, absolutely wrong. The functor 'wholly' distributes both over conjunction and over disjunction. So do its synonyms, such as: 'completely,' 'entirely,' 'altogether,' 'utterly,' 'one hundred percent.' Therefore, P.A.E.M. means that any fact or situation is either wholly true or else wholly false. The functor meaning whole falsity (which we'll write as '$\neg$' and read as 'It's wholly false that' and the like, 'It's not at all (not in the least = nowise by no means) the case that') fulfils DeMorgan laws: $\neg(p\wedge q)\leftrightarrow.\neg p\vee\neg q$, and $\neg(p\vee q)\leftrightarrow.\neg p\wedge\neg q$, since $\neg p\leftrightarrow HNp$ (where 'H' means 'It's wholly true that'). P.A.N. is thus written: $HN(p\wedge Np)$, which is equivalent to $\neg(p\wedge Np)$, which in virtue of De Morgan yields $\neg p\vee\neg Np$: either it's wholly false that p or else it's wholly false that it fails to be true that p. In virtue of involutivity of simple negation, 'N', and of the (definitional) equivalence between '$\neg$' and 'HN', «$\neg Np$» is equivalent to «Hp»: thus, P.A.N. is nothing else but another formulation of P.A.E.M. More interestingly: let 'L' mean '(at least) to some extent (or other, however small)' or any synonym thereof such as 'more or less.' Then we obviously have: $\neg p\leftrightarrow NLp$ (we can define «Lp» as abbreviating «$NHNp$»). Hence, «$\neg p\vee\neg Np$» is equivalent to «NLp∨NLNp», which by De Morgan is equivalent to «N(Lp∧LNp)», which is an alternative formulation of P.A.N. Of course we also have the equivalences «N¬p↔¬¬p», «LHp↔Hp», «HLp↔Lp» and so on (those three functors, '¬', 'H' and 'L' being intrinsically two-valued).

Many instances of P.A.N. and of P.A.E.M. (and of their equivalents) may be shown to fail. Let's see some of them: Either Albert Dürer was altogether handsome, or he was not handsome at all; either horse-cars are one hundred percent rapid or else they are not rapid at all; no one at all is such that, while he speaks more or less stammeringly, he also to some extent fails to speak stammeringly; it's nowise the case that some people are both more or less educated and more or less uneducated; whenever the weather is wet in some degree, however small, it completely fails to be dry (supposing 'wet' abbreviates 'not dry' or conversely); nothing at all is more or less useful if it is, to some extent however small, more or less useless; either a punishment is utterly deserved, or else it's utterly undeserved; any account is either one hundred percent true or else one hundred percent untrue.

The above arguments rely on some assumptions (like the distributivity of 'wholly' into disjunction, or the validity of De Morgan laws for strong negation, i.e. for 'not … at all') that admittedly can be challenged — although I'm aware of no discussion about the former while controversies about (some of) the latter take place in discussive contexts — intuitionism and the like — which have but little in common with our enterprise. Anyway, those assumptions look, to myself at least, quite correct and are possessed of high plausibility.

My conclusion is that, if we need, in order to accept real fuzziness or graduality — the fact that many properties come in degrees — to keep clear of unwanted principles like P.A.N. and P.A.E.M., this is no ground for rejecting simple excluded middle or noncontradiction, which, quite on the contrary, are equivalent situations in the sense that every fuzzy situation strictly implies the corresponding instance of excluded middle and noncontradiction.

§3.— The case for fuzziness and the issue of ontological neutrality

One of the objections most commonly levelled against fuzzy-set approaches is that whatever can be accounted for via fuzzy sets can also be described and explained in classical, nonfuzzy, ways. Oddly enough, many fuzzy-theorists agree with that objection, only they reply that, that being the case, fuzzy accounts have as great a right to be used as classical accounts do; as great and perhaps more, if they can simplify scientific theories.

Nevertheless, that reply is unconvincing and evinces a merely half-hearted defense of fuzziness. For, if there are theoretical advantages in using (and hence — I guess — in holding) a fuzzy approach, then there are grounds to say that fuzzy approaches are truer to reality. This, after all, is how theories get (relatively) justified as reflecting reality. Likewise, even though two theories can explain all physical phenomena, if one of them is all-in-all simpler — or, more generally, if it's fraught with less epistemologically unwelcome features — then it is not arbitrary to think that it's more in agreement with reality than the other. (Needless to say, some theory may possess epistemologically greater advantages from a certain outlook or background, while a rival theory turns out to be preferable from a different outlook. Justification is always relative to some background or viewpoint. But this nowise makes truth relative. Quite the contrary happens: (relative) justification of a theory is justification of the belief that the theory is true, true in a nonrelative sense.)

Moreover — and still more importantly — if there really are either facts (states of affairs) or sets, it cannot be a matter of mere choice whether a fuzzy approach or a classical one is preferable (even if it might be a matter of choice whether we believe that the former or the latter is preferable). For, what a classical approach does is replace fuzzy predicates, like «being tall,» with nonfuzzy ones, like «being six feet tall» (supposing it's nonfuzzy, i.e. supposing that, if a man is, say, 71 inches tall, then it's utterly false to say that he is six feet tall). Now, if there really are facts, it can be asked whether someone's being tall is a fact. Were fuzzy approaches to be rejected, no such fact could exist, all facts being then expressed through sentences whose predicates were nonfuzzy. Likewise, if there are sets we can wonder whether richness is one of them; rejecters of fuzzy approaches are bound to hold that richness doesn't exist (the only sets that could then exist would be of the kind: owning one thousand dollars, supposing it to be nonfuzzy, i.e. to be such that whosoever has $ 1,001 nowise belongs to the set under consideration).

If we want to take logic as ontologically neutral — which I do not — then we're faced with a dilemma. If we choose classical logic, we're ruling out some ontologies (the ones that postulate either fuzzy sets, or facts expressible through sentences containing fuzzy predicates, or both). If on the other hand we choose a *meagre fuzzy logic*, we don't rule out a classicist ontology (since such logics don't lay down that *there are* fuzzy truths, they only fend for the epistemically possible case wherein *there might be).* In fact, a meagre fuzzy logic is set up by dropping certain classical laws for negation (the PseudoScotus principle and whatever belongs with it, like disjunctive syllogism), while adding a one-place *overaffirmation functor* (meaning 'altogether'), such that the string of an overaffirmation functor followed by a negation functor is an overnegation functor, for which all classical properties hold. (The last sentence should be qualified as regards involutivity, which holds for overnegation only in that, for any «p», «p» and «It's wholly false that it's wholly false that p» are interchangeable or *congruent* when the only functors taken into account are classical, i.e. when weak or natural negation is left off.) What a classicist ontology may add is the further claim that, for any «p», its failing to be the case that p is equivalent to its utterly failing to be the case that p; a further claim that meagre fuzzy logics don't forbid, even though they don't enforce it either — which, instead, classical logic does. So, classical logic is less ontologically neutral than meagre fuzzy logics.

On the other hand we may strengthen a meagre fuzzy logic into a *plentiful fuzzy logic* by laying down postulates saying that there are fuzzy situations — situations, that is, neither wholly true nor wholly false; e.g. by introducing as primitive symbol an equivalence functor 'I' and by stating that every selfequivalence is as true as false (as we concluded above, Sect. 1). Then we cannot further strengthen the simple negation 'N' by laying down «$HN(p\wedge Np)$» without incurring overcontradiction, i.e. countenancing a formula of the form «$p\wedge HNp$», i.e. «$p\wedge\neg p$» — and, even if simple contradictions may be acceptable and even true, all overcontradictions are utterly false, so that every system countenancing overcontradictions is bound to be deliquescent, i.e. absolutely inconsistent (see section 4 below, where those notions are duly explained). If a meagre fuzzy logic is more ontologically neutral than classical logic, a plentiful fuzzy logic is instead as little neutral as the latter, only in an opposite direction.

Anyway, ontological neutrality is a matter of degree. Some people fancy that logic is ontologically neutral provided it doesn't assert any existence at all. (So, in order for classical predicate-logic to become neutral it only needs to drop principles such as «If everything is such that…, then something is such that …,' and the like.) That is a delusion. For you get ontologically entangled or committed either by stating something's existence *or* by *denying it.* Now, almost all logical (quantified) statements are of the form «Everything is such that …, i.e. «There's nothing such that not…»

(Don't say that, since what purportedly is denied existence by logical stipulation is impossible, it's nothing, and then there's nothing such as «its» existence to be denied. For a different logic may regard as possible something that the logic we were holding regarded as impossible. And anyway we can take up the formal (metalinguistic) way of speaking, and, instead of saying that we deny something's existence, say that our theory contains some statement of the form «Nothing is such that …») People have become so worried about positive existential commitments that they neglect as harmless any negative existential commitment. But would a nihilist logic, stating that nothing exists, be ontologically uncommitted or neutral?

Were we looking for a completely neutral logic, I think we'd be bound to choose a «logic» without tautologies or rules of inference, as Bochvar's internal system, since such a «logic» would rule nothing out; it would allow things absolutely worthy as well as absolutely unworthy at the same time.

This is why I'm not prepared to allege ontological neutrality in support of nonclassical logic. Should someone think that he had sound reasons for precluding fuzzy ontologies, he'd be justified in cleaving to classical logic. (Only he should be aware of his position's being ontologically committed.)

My own position is that fuzzy ontologies are much more plausible than classical ones. The issue of course arises only if there are either sets or facts. (I myself think there are both the ones and the others; be that as it may, let's regard the issue as arising on some ontological hypothesis.) My reason for thinking that some fuzzy ontology is correct is not that we couldn't in principle replace our fuzzy statements by nonfuzzy ones; even though such a replacement is practically unfeasible, in principle we could make it, were we omniscient and were there no fuzzy facts or sets at all. Nor is my reason some hidden verificationism, yielding the conclusion that, since we cannot in fact always verify crisp replacement-sentences called to take the place of fuzzy statements, there really are uncrisp situations. (Notice, though, that that kind of verificationism would be much more defensible than the one that rules out as nonexistent whatever cannot be verified; what the verificationism here considered would state is that, when we can verify some kind of situations while we cannot always verify another, related, kind of situations, then it's not the case that the only real kind of situations is the latter.)

My real grounds for thinking that there probably are fuzzy situations are the following ones:

1. Positing fuzzy predicates usually simplifies our theories in most scientific fields, including philosophy (epistemology as well as ontology).
2. The cleavage between customary (pre-theoretical, «intuitive») ways of thinking and speaking, on the one hand, and scientific ones, on the other, would become unbearable without fuzzy predicates and statements, the loss of which would bring about such a splitting of our whole thought that thenceforth science could hardly keep its roots in pre-scientific convictions and evidences.
3. Many of those pre-scientific ways of speaking and thinking, which need to use fuzzy predicates, are much more plausible, and give us a much more attractive and cohesive worldview, than their crisp counterparts. (Thinking of evolution without fuzzy predicates seems an uninviting enterprise, since what we'd have, instead of chains of degrees of some properties' exemplification, would be chains of juxtaposed exemplifications of entirely disjoint properties.)

4. All talk about degrees would be precluded if crisp predicates alone were legitimate. Along with degree-talk, use of comparative constructions would also fall victim to our rarefication policy. For, instead of stating, say, that milk is more nourishing than rice, we should say that milk exemplifies some property (having, e.g., so many calories), whereas rice exemplifies some other property, and that the property exemplified by milk stands in some relation to the one exemplified by rice, which relation must be definable without resorting to degree-talk — as, e.g., the *greater-than* relation among numbers happens indeed to be. However, degree-talk and comparative constructions take so huge a share of our most basic worldview that I cannot help regarding their sacrifice as most untoward and unwanted.

5. Some fuzzy approaches make it possible formally to develop valuable philosophical intuitions held in the Platonic and Neoplatonic tradition, like the following ones: the contention that there are many degrees of truth and reality (which, incidentally, may afford a solution to the problem of nonexistent objects — fictional characters and the like — less difficulty-ridden than current classicist solutions); a view of modality which allows for degrees of possibility — and hence also of necessity — thus making it sensible to say that a fact is both (to some extent) contingent and yet also (to some extent) necessary; and a view of God as exemplifying all properties — each of them insomuch as the property exists — which, besides answering more faithfully than classical accounts to the ontologically unifying or cementing task God is called upon to discharge in any philosophico-theological approach helping to overcome scattering or splintering viewpoints about the world or about science, and being able to solve in a pretty straightforward way the well-known paradoxes of philosophical theology, is more in line with our general physicalist worldview than spiritualist classical theology which rejects all contradictions as absurd.

6. Another fruitful field where use of fuzzy approaches seems promising is the existence of moral dilemmas, the framing of a fuzzy deontic logic recognizing conflicts of values and of duties and, accordingly, accepting that two contrary actions may both be obligatory, even if — in most cases — one of them is more so than the other.

7. Fuzzy approaches alone are liable successfully to grapple with several philosophical puzzles, as Unger's point about a body's boundaries (see [41]), or problems of identity and spatio-temporal continuity, all of which may be solved if we regard bodies as fuzzy sets of their respective parts.

8. Some fuzzy approaches can afford a winsome way of dealing with Zeno's paradox of the arrow and similar issues, by stating that, even though at any lapse included in the total duration of its movement the travelling body both is and is not in any stretch encompassed by the whole span it will have traversed, still at any such lapse the body is more in some such stretches, less in others, the difference being greater the greater are the extent to which the body is in the former stretch under consideration and the distance between the two stretches — supposing them to be nonoverlapping stretches at least as large as the travelling body. Such kind of solutions would be welcome, since there is no really satisfactory classical solution to the paradox of the arrow.

9. Some fuzzy approaches may also help to cope with some paradoxes surrounding time-flow: they are ensuant upon degrees of simultaneousness: a temporal logic wherein some lapse or event may be said to be more, or less, simultaneous with another than some third lapse or event is may constitute a valuable alternative to classical temporal logics.

10. Some fuzzy approaches may help to solve naive set-theoretical paradoxes, even though I feel sure some classically received device or other — like ultimate classes and stratification — is to be resorted to in order to prevent remaining paradoxes. Nevertheless, naive paradoxes, which can be accepted as true contradictions in an axiomatized fuzzy set-theory, are not epistemologically on a pair with «perverse» paradoxes, which don't occur in ordinary thought. Thus, even if recognizing fuzziness doesn't dispense us from falling back on some additional barrier against new set-theoretical aporias which show up in an axiomatized fuzzy set-theory, still the fuzzy move is a legitimate one, since it extends the boundaries of such naive set-theoretical ground as can be kept in an axiomatized — hopefully coherent — set-theory.

11. Fuzzy approaches may also afford a way of coping with epistemological issues in a suppler way than classical accounts. Take for example the canvassing of data-candidates in a coherence theory of truth like the one developed by Rescher: if the theory is overconsistent — i.e. flatly rejects all contradictions (see section 5 below)— and consequently rules out *objective* degrees of truth, then, once the sifting process has taken place, the candidate is either unqualifiedly and utterly shut off or else accepted as downright true, whereas a gradualistic approach may assign to the sifting process's outputs — not just to its inputs — several degrees of truth — not of mere plausibility — in function of a number of parameters. Thus, problems brought about by conflicting criteria about the application of a predicate, by the existence of two discording legitime extrapolations and the like can be handled in a flexible way that, in function of other factors, may recognize to one of the two contradictory claims a larger truth-degree without thereby regarding the other claim as entirely false.

I've gone into all those issues in other papers, mentioned among the References below. I take it to be fine and advantageous that by adopting a fuzzy logic we can deal with all those issues in a far clearer, simpler and more straightforward way than is possible within a classical framework ruling out objective fuzziness. For, a logical revolution is not worthwhile unless the range of problems successfully dealt with thanks to the new logic one espouses is large and encompasses problems belonging to sundry fields formerly regarded as unrelated or only loosely connected, thus grounding a more cohesive, less ad hoc kind of solutions than used to be entertained.

I ought to admit that the foregoing considerations rest upon assuming that such surface-structure formulas as contain, embedded within the formula, some occurrence of any extent-symbol result from a deep-structure formula wherein the extent-symbol occurrence is prefixed to the remainder of the formula. 'Andrew is very shrewd' is a surface-structure transform of a sentence like 'It's very true that Andrew is shrewd'; 'William is somewhat stubborn' is the transform of 'It's somewhat true that William is stubborn.' Likewise, comparative constructions are of the form «It's less true that p than that q.» So, e.g., 'Roger is glibber than Ted' is a transform of «It's less true that Ted is glib than that Roger is glib.' (And there is no vicious circle here, since «It's less true that p than that q» abbreviates: «That p implies that q and it's wholly false that that q implies that p.») Those assumptions can be bolstered up with telling arguments, the most persuasive thereof being that the equivalence is intuitively appealing and gives a simple, clear account of many adverbial phrases. A next argument is that other accounts of those phrases don't carry conviction, all of them being either clumsy and too intricate for words, or else farfetched. (Moreover, the grounds alleged in support of those accounts often confuse semantics with pragmatics.)

§4.— How is classical logic to be weakened?

Our foregoing discussion leads us to conclude that there are reasonable grounds for thinking that there is fuzziness in things, that is to say: that there really are fuzzy situations — or, if you want to put it that way, that fuzzy states of affairs do really obtain in the world. Accordingly, our logic should rather allow for fuzziness, instead of ruling it out beforehand. Now, when we go about setting up a meagre fuzzy logic, we want to have a logic which, while admitting of degrees of truth, should otherwise be as strong as possible, its functors departing from their classical counterparts as little as is required in order to let fuzziness in.

As I said above, what we need to jettison, as regards simple (i.e. natural) negation — the mere 'not' — is Pseudo-Scotus along with disjunctive syllogism. All arguments mustered in order to prove that any contradiction entails anything (and hence that any negation-inconsistent system is deliquescent — i.e. *absolutely* inconsistent) are fallacious, since they all assume disjunctive syllogism. (See [35]: 321ff., [17], [12]: 337-8.)

The principles which are to be kept as regards disjunction, conjunction and natural negation are, to my mind, the following. (Remember that '$\vee$' is disjunction; '$\wedge$' conjunction; 'N' natural negation.)

A theory is an ordered quartet $<V, F, T, R>$, where *V* is a set of primitive symbols, *F* (the set of wffs) is a proper subset of the set of strings of members of *V*, *T* (the set of theorems) is a subset of *F* which is closed for each member of *R*, and *R* is a set of deduction-rules, each $r \in R$ being a set of pairs $<\Gamma, r>$ where Γ is a set of wffs (premises) and r a wff (the conclusion). (For simplicity sake we'll write a deduction from $\{p_1, \ldots, p_n\}$ to «q» as: $p_1, \ldots\ p_n \vdash q$.) A theory $\mathcal{T}$ is said to contain or *validate a deduction* from a set of wffs Δ to a wff «p» iff there is a chain of deductive steps beginning with Δ and ending in «p», where every *deductive step* belongs to a deduction rule of $\mathcal{T}$ and is such that each premise in it is either a member of Δ or a theorem or a conclusion reached in a previous deductive step in the chain. If a theory $\mathcal{T}$ validates a deduction, the latter will be said to be a deduction of $\mathcal{T}$. Within a theory $\mathcal{T}$ two wffs «p» and «q» are *interchangeable* iff for any wff «r» containing occurrences of «p», $\mathcal{T}$ validates the deduction from «r» to the result of replacing in «r» any number of those occurrences of «p» with occurrences of «q», and conversely. For any «p», «q», «r», a theory $\mathcal{T} = <V, F, T, R>$ is *well-behaved* iff: $\vee$, N, $\forall$x, $\exists$x $\in$ *V* ('$\forall$x' being the universal quantifier while '$\exists$x' is the existential quantifier, 'x' standing for *any* individual variable); and for any wffs «p», «q», «r», and for any variable «x» the following hold:

(i) $p \wedge q \vdash p$ is a deduction of $\mathcal{T}$;

(ii) $p \vdash p \vee q$ is a deduction of $\mathcal{T}$;

(iii) $p \vee q \wedge r$ and $r \wedge q \vee . r \wedge p$ are interchangeable;

(iv) p and $q \vee p \wedge p$ are interchangeable;

(v) The functor 'N' possesses these characteristics:

(1) $\ulcorner p \vee Np \urcorner \in T$;

(2) $\ulcorner p \urcorner$ and $\ulcorner NNp \urcorner$ are interchangeable;

(3) $\ulcorner N(p \wedge q) \urcorner$ and $\ulcorner Np \vee Nq \urcorner$ are interchangeable;

(4) $\ulcorner \exists xp \urcorner$ and $\ulcorner N\forall xNp \urcorner$ are interchangeable.

A well-behaved theory $\mathcal{T}$ is said to be *inconsistent as regards some negation-functor* thereof, 'N', which satisfies all properties above iff, for some «r», «r» and «Nr» are theorems of $\mathcal{T}$. A theory is said to be *simply* (or negation) *inconsistent* iff it is inconsistent as regards some negation-functor thereof. A theory is said to be *contradictorial* iff, in addition to being simply inconsistent, it validates adjunction, to wit: p, q $\vdash$ p$\wedge$q. Consequently, any contradictorial theory contains some theorem of the form «p$\wedge$Np». Since contradictorial theories are well-behaved, they also contain the principle of noncontradiction «N(p$\wedge$Np)» and thus they contain infinitely many contradictions of the form «q$\wedge$Nq$\wedge$N(q$\wedge$Nq)».

A theory $\mathcal{T}$ is said to be *deliquescent* (or trivial, or absolutely inconsistent) iff every wff of $\mathcal{T}$ is a theorem of $\mathcal{T}$, i.e. iff $T = F$. A theory is said to be *solid* (or coherent) iff it is not deliquescent. Let an extension $\mathcal{T}'$ of a theory $\mathcal{T}$ be called a *stark extension* of $\mathcal{T}$ iff every deduction validated by $\mathcal{T}$ is also validated by $\mathcal{T}'$. A theory $\mathcal{T}$ is said to be *overconsistent* iff every stark extension of $\mathcal{T}$ which is inconsistent as regards some negation-functor of $\mathcal{T}$ is deliquescent. Classical (i.e. two valued truth-functional) logic is overconsistent. So are Łukasiewicz' systems, intuitionistic logic, Gödel's systems and most other systems of logic.

A theory is said to be *paraconsistent* iff it is not overconsistent. Several interesting systems of logic are paraconsistent: Routley's and Routley & Meyer's systems of relevant paraconsistent logic (see [38], [39]); Kotas & da Costa's generalized Łukasiewiczian logics ([16]); my own systems *A* (see [21] through [29]); da Costa & d'Ottaviano's system ([19]); G. Priest's system LP ([36]); Sobociński's 1952 system (in [40]); other logics are quasi-paraconsistent — they fail to be full-fledgedly paraconsistent only in their failing to satisfy some among the well-behaved-ness principles laid down above; so, e.g., da Costa's systems *C* fail to accept involutivity, De Morgan and Noncontradiction, even though they accept Excluded Middle (as well as the converse to the intuitionistic half of involutivity, viz: «If NNp, then p»). (See [6], [7].) Related systems have been proposed by Arruda ([1], [2], and elsewhere), by Asenjo ([4]), by Batens ([5]). (For a survey, see [3] and also [11], where the author N. Grana proposes his own system IDL which is both paraconsistent in a wide sense and intuitionistic.)

The foregoing account of negation is based on regarding negation as having the properties of the one-place operator of a De Morgan (or quasi-Boolean) algebra, which properties are in fact possessed by negation not only in classical logic, but in quite a few many-valued logics as well (e.g. in Łukasiewicz'), and by the validity of simple noncontradiction and simple excluded middle — this in virtue of considerations brought forward below, in section 5.

Many-valued logics may be contradictorial and hence well-behaved. All depends on their choosing as designated or true an appropriate subset of the set of truth-values. A three-valued logic whose values are 0, ½, and 1 can be contradictorial if its designated values are both ½ and 1. An infinite-valued logic whose truth-values' set is [0,1] can be contradictorial if its designated-truth-values' set is [½ , 1]. Choosing a set of designated truth-values is not nugatory or arbitrary, since thereupon hinges what statements one is going to countenance as true and even as logically true.

Besides the aforementioned functors, any system aiming at capturing intuitive ideas about fuzziness needs to contain functors of degree, including an overaffirmation functor, 'H', to be read 'It's wholly true that'. A well-behaved theory $\mathcal{T}$ will be said to be *upright* iff it contains a one-place functor 'H' such that for any «p» and «q» and for any variable «x»:

(1) ⌜NHp⌝ and ⌜HNHp⌝ are interchangeable;

(2) ⌜H(p∧q)⌝ and ⌜Hp∧Hq⌝ are interchangeable:

(3) ⌜H(p∨q)⌝ and ⌜Hp∨Hq⌝ are interchangeable;

(4) ⌜p⌝ and ⌜p∨Hp⌝ are interchangeable;

(5) ⌜H∀xp⌝ and ⌜∀xHp⌝ are interchangeable;

(6) ⌜H∃xp⌝ and ⌜∃xHp⌝ are interchangeable;

(7) The following deduction is validated by $\mathcal{T}$: p∨q, HNp ⊦ q

Classical logic is an upright system wherein, for any «p», «p» and «Hp» are interchangeable (or, which boils down to the same, — for any «p», «Np» and «HNp» are interchangeable).

We can strengthen an upright system of logic remaining short of classical logic, by adding the *Strong Excluded Middle*, S.E.M. for — short, viz. that for every «p»: (S.E.M.) «p∨HNp» is a theorem of $\mathcal{T}$.

An upright system wherein every instance of S.E.M. is a theorem will be said to be *straight.* (Classical logic is of course straight. But a system of logic can be straight without being classical.)

Any straight system can be regarded as a conservative stark extension of classical logic, the sequence 'HN' standing for classical or strong negation (overnegation), while simple negation ('N') is a nonclassical functor. Classical logic is all right whenever what is alone taken into account is a sentence's being either wholly false or not wholly false.

We can now differentiate between simple contradictions (or antinomies), formulae of the form «p∧Np», which may be true (up to a point, namely: at most half true), on the one hand and *overcontradictions*, formulae of the form «p∧HNp» which are always completely false and even absurd.

Every straight system validates all deductions belonging to what can be called *the rule of endorsement*, to wit:

(R.E.) NHNp ⊦ p

What R.E. says is that whatever is more or less true is true, period. Thus, if our set of truth-values is [0,1], R.E. will enjoin our choosing]0,1] as the set of designated truth-values. Still, we obviously want, for any formula «p», and any valuation *v*, to regard *v*(∀xp) as the *infimum* or g.l.b. of the functional values into which valuations that are x-variants of *v* map the argument «p» — i.e. as the g.l.b. of {u: for some x-variant valuation of *v*, *v*', *v*'(p)=u}. The goal thereof is to secure that «∀xp→p[x/y]» should be valid. But this self-recommending policy will give rise, if we cleave to the real interval [0,1], to what will be called ω-over-inconsistency, to wit that a formula «p[x]» may be such that, for every individual constant 'e', «p[x/e]» is true while «∀xp» is wholly false. We are faced with the option of either rejecting S.E.M. or choosing a set of truth-values such that the infimum of the values different from zero should be different from zero. This can be done by resorting to hyperreal truth-values, as I'll show below, in section 7.

§5.— Rejecting the maximality rule and defending simple excluded middle

We can deal with fuzziness without resorting to S.E.M. What definitely must be waived is the maximality rule, viz.: p ⊢Hp. Such a rule, though, has been enforced not just in classical logic, but in most nonclassical logics, including Łukasiewicz'. The purported rationale for it is that nothing can be rightly asseverated unless it's quite true, true without mixture of falsity. So, 'true' *tout court* would be equivalent to 'utterly true.' But to my mind such a reason doesn't carry conviction. For, when we assert some sentence, we to be sure regard it as true; but why on earth should we regard it as wholly true? When I say that I'm hungry, I'm not saying that I'm altogether hungry, but just hungry. In order for someone to be clever he needs just to be clever, nowise to be altogether clever. The maximality rule thus leads to relinquishing what is of top-importance in fuzzy logic, namely: truth-nuances, in virtue of which not all that is true is entirely so; in other words, the intuitive idea that truth (and existence) comes in degrees — in fact, most ordinary properties do come in degrees.

If a system both contains the maximality-rule and is upright, it's a Boolean logic, since, in order for «p∨Np» to be true, it would need to be completely true, which could be the case only if 1 (or <1,1,…,1>) were the supremum (the l.u.b.) of |p| and |Np|, whichever they may be; therefore in virtue of the other requirements, |p∧Np| will be 0 (or <0,0,…>) for every «p». The only such logic is hence classical logic.

Now, once the maximality-rule is waived, nothing can restrain us from holding noncontradiction and excluded middle, even though we countenance fuzziness. By taking [½,1] as the set of designated truth-values we'll be able to secure both goals.

For, in virtue of the choice, we'll regard as true *simpliciter* whatever is at least half true. But of course out of any pair of sentences such that one of them is the other's (simple) negation, one at least is half true, and one at most is more than half true. So, the disjunction of them will perforce be at least half true, while their conjunction will be at most half true; accordingly, that conjunction's negation will be at least half true, too. Therefore, both principles are secured. In order for either to be a correct principle it needn't be entirely true, but just true. Consequently, those who (like Machina in [18]) reject simple excluded middle on the basis that, when neither |p| nor |Np| are not completely true, |p∨Np| cannot be completely true either, peremptorily take it for granted that asserting a sentence is tantamount to vouching for its complete truth, which assumption we are just criticizing.

The foregoing remarks may be mistaken for purely technical considerations, which they are not. In fact, what they are aimed at showing is that every instance of noncontradiction, as well as every instance of excluded middle, cannot but be an objective truth (a real fact). In order for any instance of excluded middle to be the case it's required that the disjunction of the two mutually contradictory states-of-affairs should obtain. But it cannot but obtain, — for else the world would be alethically undetermined — i.e. would contain truth-value gaps. It may be possible that both mutually contradictory states-of-affairs obtain; what by no means can be the case is that neither should obtain at all. Likewise, every contradiction must be false; of course, not necessarily quite false, but just false: the world's set of objective truths fails to contain any contradiction, its failing to contain any given contradiction being at least half true, and so true, but not always completely true.

Were we to identify objective or nonsemantic truth (truth of propositions or states of affairs, as opposed to sentential truth) with existence or reality of those states-of-affairs (an identification which can be soundly argued for; see ([33J), then what we should say is that there objectively are degrees of reality, and every contradiction is at most half real, while every instance of excluded middle is at least half real.

§6.— Why being more or less true entails being true

As we've seen, resorting to S.E.M. is not absolutely necessary for coping with fuzziness. What is indispensable is to accept simple excluded middle and simple noncontradiction (since, as we saw above, the very statement of fuzzy situations entails the corresponding instances of excluded middle — and, thus, of noncontradiction, too). Nevertheless, I think that S.E.M. should also be recognized as a valid principle. Along with it strong noncontradiction (S.N.) ought to be accepted, as well: S.N. is the principle «N(p∧NHp)», for any «p», which should be read as follows: «It's not the case that: p while it's not wholly true that p.» S.N. can very easily be shown to be equivalent to S.E.M. My reasons for holding S.E.M. (and hence S.N., too) are the following:

1. It seems to me quite arbitrary to regard as a cut-off point the half-true-half-false truth-value. Why on earth should we refrain — when we speak contextlessly, at least — from asserting a sentence whose truth-value is 0.4999999998?

2. Our discussion of simple noncontradiction and simple excluded middle lead us to argue, in support of them, that they need perforce to be true, even though they are not quite true. Our natural reply to those who complain that many instances thereof are not altogether true was that being just true is something different from being altogether true. But exactly the same considerations can be put forward in support of S.E.M. and S.N.: what can be said is that they cannot but be — to some extent or other however small — true, just true; even if, or course, most instances thereof are highly false as well, none is altogether false.

3. That 'true,' *tout court*, means something different from 'at least half true', as well as from any other similar qualification, can be also shown by pointing out that prefixing some qualification or nuance-functor to 'true' may result in something very different from prefixing it to, e.g., 'at least half true.' Thus, for instance, 'somewhat true' nowise means the same as 'somewhat at least half true.'

4. Just as we say that all healthy horses are fast, without thereby committing ourselves to holding that all healthy horses are fast to the utmost, we so don't thereby commit ourselves either to holding that all healthy horses are at least half fast. Our statement wouldn't become completely untrue were we to find out that some healthy horses are only forty or thirty percent fast. And, by the same token, it wouldn't turn out wholly untrue either should there be healthy horses only twenty-nine percent fast. And so on. What would alone render completely false our statement would be there being healthy horses which were not fast at all. Accordingly, our saying that some sentence (or some proposition, or state-of-affairs, if you like) is true will not completely lack truth unless the sentence in question is not true at all. Since what is required in order for a statement to be right is nothing else but its being true (just true, without further qualifications), and any statement is (to some extent or other, however, small) true unless it's wholly untrue, we can safely state any sentence provided we are convinced it's not altogether false. (Of course, pragmatically there are, in most contexts, further requirements to be met.)

5. Should only at-least-half-true sentences count as true *simpliciter*, we'd be at a loss as to how to frame our *mere* conditional functor meaning 'if … then'. I think that no system of

logic S is *proficuous* unless it's a conservative extension of classical positive calculus, which entails that S needs to keep all classical tautologies wherein there's no occurrence of negation. (The ground for that requirement is that, insomuch at least as what prompts us to go beyond classical logic is fuzziness, we find no fault with the set of positive classical tautologies. In other words: whenever simple or natural negation is not involved, our philosophical remarks, which hinge on recognizing objective, real fuzziness or graduality in the world — degrees of truth or reality — don't clash with classical logic. True, some people — the relevantists — carp at classical conditional tautologies for quite other reasons. But that has nothing to do with our own motivations.) Hence, if S is proficuous, any formula «p» of S containing as its only logical symbols conjunction, disjunction, and the mere conditional will be a tautology of S iff it's a classical tautology. We can now see that any proficuous system is straight (and thus enforces S.N. and S.E.M.) iff its mere conditional functor is defined like this: ⌜p⊃q⌝ eq ⌜HNp∨q⌝ (in words: «p only if q» abbreviates «Either it completely fails to be the case that p, or else q»). Since there's a strong rationale for proficuousness, and proficuousness is equivalent to straightness (on the base of the definition above), the rationale can bestow plausibility upon straightness. On the other hand, the definition itself is most likely correct, since: 1) it tallies with our previous comment that classical logic is all right provided all that is taken into account is sentences' being either wholly false or not wholly false — so that-p's entailing that-q is nothing but that-p's being wholly false or that-q not being wholly false (i.e. being just true, to whatever extent); 2) it shows that *modus tollens* is a notational version of disjunctive syllogism. Should we take the alternative course of choosing a Gödelian conditional-functor, along with a designation-policy which would block S.E.M. — which is what Gödel did — our ensuing system wouldn't be proficuous, because of the failure of Peirce's law (see remark number 8 below). But any other alternative would be still worse for a number of reasons. Notice that the Gödelian conditional can be defined within a straight system containing implication 'D' as a functor as follows: «p only-if (in Gödelian sense) q» abbreviates «p⊃q∨NHN(p→q)», where — remember — the implication functor is such that, for any «p» and «q» |p→q|╪0 iff|p|≤|q|.)

6. S.N. can also be shown to be right by pointing out that Reality is, to some extent or other, incompatible with contradictions. So, whenever Reality includes some contradictions, it does also exclude it, its both including-and-excluding it being possible by the fact that the contradiction is neither wholly included nor wholly excluded by Reality, but it is both included and excluded to some extent only. Now, for any «p», its failing to be wholly true that p is, in some degree or other, incompatible with the fact that p (otherwise it could be both altogether true that p and yet altogether true that it's not wholly true that p — i.e. wholly false that it's altogether true that p — which is obviously absurd, since it is an *over* contradiction). Therefore, «p and it's not wholly true that p» is a contradiction of some sort, since it's a conjunction whose conjuncts are, to some extent or other, incompatible with one another. Consequently, such a conjunction is, to some extent or other, excluded by Reality. So, its failing to be the case cannot but obtain, at least in some degree.

7. Unless we regard as designated or true all truth-values except *zero*, we'll face the following anomaly. Let's introduce into our system a one-place functor 'ξ' such that, if |p| is designated, |ξp| = |p| and else |ξp| = 0. Of course we'll then validate the deduction p ├ ξ. Still, we won't have the theorem — schema «p⊃ξp». And then the metatheorem of deduction will break down. Nevertheless, it can reasonably be demanded that the

metatheorem of deduction should fail *only* when either universal generalization or a Gödelian rule plays a role in the deduction. (By *a Gödelian rule* I understand a rule of the form p ⊦ μp, where 'μ' is a one-place symbol such that, for some «p» and «q», μ(p∨q) ⊦ μp∨μq is not a valid deduction.)

8. Unless we regard as designated all truth-values except zero, we cannot secure the validity of Peirce's law («p⊃q⊃p⊃p», for any «p» and «q»). (The Gödelian many-valued logics fail to recognize that law.) That would be a pity, since it is a classical positive law, thus no fault is to be found with it. Hence, if we want our system to be proficuous (let alone straight) we need a designation-policy which renders S.E.M. valid.

Some of the foregoing remarks rely upon assuming that, unless we adopt ½ as the cut-off point (the assertability threshold), we ought to accept as designated or simpliciter true all values greater than zero, i.e. to recognize as assertible whatever is not utterly false. Nonetheless other, lower, candidates might be brought forward, say $(½)^{\surd 2}$, which is the least pretty true value, supposing a fact or value to be *pretty true* in case it is by no means much more false than true, i.e. its being very false implies its being true, where |It's very true that p| = $|p|^2$. (I'm supposing our value field is the real interval [0,1].) Such candidates can be accepted as thresholds of pragmatically warranted assertibility in some kind of communicational contexts or other, but as thresholds of purely semantic assertability they are fraught with the same difficulties besetting ½.

The foregoing discussion's upshot is that S.E.M. and its equivalent, S.N., indeed hold (to some extent or other). Their holding yields R.E. (the Rule of Endorsement) via disjunctive syllogism for strong negation. In fact, S.E.M. can be derived from the weak excluded middle (viz. «HNp∨NHNp»: either it's wholly false that p or else it isn't wholly false that p) via R.E. So R.E. and S.E.N. are equivalent, in the sense of following one from the other. For the weak excluded middle is the least almost everyone is likely to concede — it is even a theorem of Gödel's infinite-valued system aiming to embody a constructivist view slightly stronger than Heyting's. (That can be shown, still more perspicuously, for weak noncontradiction — which is equivalent to weak excluded middle — viz. «N(HNp∧NHNp)», which is even a theorem of intuitionistic logic; within our present account, weak excluded middle and weak noncontradiction are equivalent in virtue of the properties of simple negation-involutivity and De Morgan.

R.E. turns out to be the key to the present approach. Some people will think that it's a principle of hyperbolism, but I'm prepared to argue that they are confusing pure semantics with pragmatic constraints. For, of course, it doesn't always suffice in order for a statement to be pragmatically apposite in most communicational contexts that it be more or less true. (Still, it is neither necessary nor sufficient that it be wholly true either). Many other requirements ought to be met. But, even if in most contexts a statement is communicationally inappropriate unless it attains some truth-degrees threshold, it remains that its being just true (to whatever extent) is sufficient for its being rightly assertible (rightly, that is, to that extent to which it is true).

What is of utmost significance is that by recognizing the validity of S.E.N. we secure Peirce's goal of keeping (a certain version of) classical logic in its entirety while expanding it, through the introduction of new functors (particularly nonclassical, simple negation), in order to deal with fuzzy or intermediary situations, which give rise to true contradictions, thus rendering simple noncontradiction and simple excluded middle somewhat false (albeit, as Peirce rightly pointed out, never downright false — indeed never more false than true). For, by designating all truth-values but zero, and by so accepting S.E.M., S.N., and R.E., we become able to have a logic which should be straight, and so a conservative extension of classical logic. This can sound odd at first blush, but it reminds us of what happened with Lewis' systems. Lewis' purpose was to weaken the Russellian conditional. In fact, though, that conditional is definitionally retrieved within Lewis' systems, even though the *strict* Lewisian conditional is weaker than the Russellian one. Hence, normal systems of modal logic are less strong than classical logic, in the sense that, by being added certain axioms, they collapse into classical logic proper, thus ceasing to be extensions thereof. Likewise, an infinite-valued logic's natural negation is weaker than classical negation, which makes those logics less strong than classical logic, even though they also are conservative extensions thereof — for certain translations only, of course.

The bearing of the foregoing remarks can be reinforced by recalling the difference between stark and nonstark extensions of a theory. The difference allows us to point out another similarity between the kind of infinite-valued logic I'm arguing for and Lewis' modal logics as regards the relation they bear to classical logic. For, Lewis' systems as well as the logic I'm propounding are stark extensions of classical logic. Instead, the intuitionistic calculus is not a stark extension of classical logic, even if it indeed is a conservative extension thereof (as was shown by Gödel in 1933, by rewriting all classical tautologies in terms of conjunction and (strong) negation (overnegation) as the only logical symbols). For, classical *modus ponens* $(HN(p\land HNq), p \vdash q)$ is not a nonsystemic inference-rule of the intuitionistic calculus. (For this issue, see ([42]: 79-80, and [13]:96-7.) If we call 'hale' any system of logic that is a stark extension of classical logic, then we'll say that the kind of logic I'm advocating is hale. (For further details about this subject, see [29].)

Furthermore, and most importantly, we need to differentiate between two ways of defining a theory, in particular a system of logic. On the one hand, we can define it as the quartet <*V*, *F*, *T*, *R*>, as I did hereinabove. But then, no grievance is after all to be addressed at classical logic, since the kind of infinite-valued logic I've been arguing for in this paper — and I'm going to sketch out in section 7 — is a stark extension of classical logic. Nonetheless our approach deserves to be looked upon as nonclassical. Why? Because another, more interesting, definition of a theory sees *V* not as a set of primitive symbols but as a set of ordered pairs <s,t>, where s is a primitive symbol and t a natural-language reading thereof. Therefore a theory may be an extension, even a conservative or even a stark extension of another theory, in the standard sense, and nevertheless a conflict may take place between the two theories, which is the case, e.g., when one of them gives some symbol a natural-language reading which the other assigns to another, therein nonequivalent, symbol. The vocabulary of classical logic as commonly held or professed comprises the pair <~, not>, which is what makes classical logic fall afoul of our arguments in support of fuzziness and against disjunctive syllogism (for simple negation) and contraposition (against contraposition involving the simple negation 'not' and the mere conditional 'only if'). An alternative classical logic, though, would replace that pair with <~, not … at all>, and then no clash would arise between such a «classical logic» and our own approach. Our differentiation, by allowing us to acknowledge sundry diverse systems having exactly the same theorems — in symbolic notation — and the same deduction rules aids us to pinpoint the focus of the discrepancy between conflicting logical approaches.

§7.— Sketching out a system of contradictorial fuzzy logic

In this Section I sketch out the *transitive* sentential calculi *Aa* and *Ap* — as well as the transitive first order predicate calculus *Aq*, grounded on the former — which capture the ideas I've been arguing for throughout the paper. Both are semantically defined, although the latter has been presented elsewhere in an axiomatized version.

To start with, we take the (standard) real interval [0,1]. For any real r such that 0≤r≤1, let the three following pairs be called *hyperreals*: {r,2}, {r,3}, {r,4}. We now introduce an order ≤ like this: if r≤r', then {r,2} ≤ {r,3} ≤ {r,4} ≤ {r',2} ≤ {r',3} ≤ {r',4}. Let such hyperreals h as {0,3} ≤ h ≤ {1,3} be called *alethic elements*. We define on the set of alethic elements these operations: if r (0≤r≤1) ∈ h, then mh={r,4}; m{1,3}={1,3}; m{1,2}={1,2}; if r∈h and n∈h (where n=2, 3 or 4), then Nh = {r',n'}, where r'= $2^{*}\log_{r}2$ (i.e. 2 raised to this power: logarithm on base r of 2), unless r=0, and then r'=1, or r=1, and then r'=0; while n'=2 if n=4, n'=4 if n=2, and n'=3 if n=3; h∧h'= *min*(h,h'); h↔h'= {½,3} if h=h', otherwise h↔h'= {0,3}; Hh = {1,3} if h = (1,3}, otherwise Hh = {0,3}; h∧h' = {rxr',3} if 3 ∈ h, h' and r∈h and r'∈ h'; h'•h = h•h' = (r×r',4} if r'≠0 and h = (r,4}, while h' is either {r',3} or {r',4}; h'•h = h•h' = {r×r',2} if r, r' ≠ 0 and r∈h and r'∈h'and either 2∈h or 2∈h'; h•h' = h'•h = {0,3} if h = {0,3}; h'•h = h•h' = (0,4} if {0,4} = h and h'≠{0,3}.

All alethic elements are said to be *designated* except <0,3>.

We now introduce our system *Aa* which is an ordered quartet <V) ¬, 1; *R*>, where *V* (the vocabulary) is a set comprising at least a sentential constant, t, and, besides: N,H,∧,•,↔,m; *F* (the set of wffs) is like this: 1) every sentential constant ∈*F*; 2) if s, s' ∈ *F*, so do Ns, Hs, s∧s, s•s', s↔s', ms; *R* is a set of just one (primitive) deduction rule: {N(NHNs∧Ns'), s} ⊦ s'. *T* (the set of theorems of *Aa*) will be defined semantically in a moment. We define a *valuation* of *Aa* as a map, *v*, carrying members of *F* into alethic elements provided for all s, s' ∈ *F*: *v*(Ns) = N*v*(s); *v*(ms) = m*v*(s); *v*(Hs) = H*v*(s); *v*(s∧s') = *v*(s)∧*v*(s'); *v*(s•s') = *v*(s)•*v*(s'); *v*(s↔s') = *v*(s)↔*v*(s'). (I hope no ambiguity or confusion arises from this twofold use of the same symbol, as standing in the left member of an equation for a symbol of *Aa* and in the right one for an operation on alethic elements.)

Now let *T* be defined as {s∈*F*: every valuation of *Aa*, *v*, is such that *v*(s) is designated}.

The intended meaning of the symbols is as follows: Np: it isn't the case that p = it's false that p; Hp: it's altogether (wholly, completely, one hundred percent) true that p; p∧q: p and q; p•q: not only but also q; p↔q: (the fact) that p is equivalent (= is true to the same extent as) (the fact) that q; mp: it's much like true that p. We now define the overnegation functor like this: «¬p» abbreviates «HNp». «¬p» is read: It's not at all (=by no means, nowise) the case that p.

The following results can be proved:

1) *Aa* is a conservative extension of classical logic;

2) Defining «p∨q» as «N(Np∧Nq)», and «p→q» as «p∧q↔p» (where '∨' means 'or', and '→' means 'implies that' or 'is true insomuch only as'), the following schemata are theorematic in *Aa*:

q∨r∧p↔.q∧p∨.r∧p

p↔p↔.q↔q

$p\wedge q\vee p\leftrightarrow p$

$p\wedge q\rightarrow p$

$p\wedge q\leftrightarrow .q\wedge p$

$p\rightarrow .p\vee q$

$p\vee q\leftrightarrow .q\vee p$

$p\wedge q\wedge r\leftrightarrow .p\wedge .q\wedge r$

$p\vee q\vee r\leftrightarrow .p\vee .q\vee r$

$N(p\wedge Np)$

$p\vee Np$

$NNp\leftrightarrow p$

$p\wedge q\leftrightarrow N(Np\vee Nq)$

$\neg(p\rightarrow q)\vee\neg p\vee q$

$p\rightarrow q\rightarrow N(p\rightarrow Nq)$ (Aristotle)

$p\rightarrow q\leftrightarrow .Nq\rightarrow Np$ (contraposition)

$N\neg p\leftrightarrow\neg\neg p$

$\neg(p\vee q)\leftrightarrow .\neg p.\neg q$

$\neg(p\wedge q)\leftrightarrow .\neg p\vee\neg q$

$p\rightarrow\neg\neg p$

$\neg\neg\neg p\leftrightarrow\neg p$

$H(Np\vee N\neg p)$

$\neg p\vee p$

$\neg(p\wedge\neg p)$

$Hp\vee Np$

$p\rightarrow q\rightarrow N(p\wedge Nq)$ (counterexample)

$p\rightarrow q\vee .q\rightarrow p$

$p\vee q\rightarrow r\leftrightarrow .p\rightarrow r\wedge .q\rightarrow r$

$p\wedge q\rightarrow r\leftrightarrow .p\rightarrow r\vee .q\rightarrow r$

$p\rightarrow(q\vee r)\leftrightarrow .p\rightarrow q\vee .p\rightarrow r$

$p\rightarrow(q\wedge r)\leftrightarrow .p\rightarrow r\wedge .q\rightarrow r$

$p\rightarrow(p\rightarrow q)\rightarrow .p\rightarrow q$ (contraction)

$p\rightarrow Np\rightarrow Np$ (abduction)

$p\wedge Np\rightarrow .q\vee Nq$ (Kleene's principle)

$p\rightarrow q\wedge p\rightarrow q$

$p\rightarrow(q\rightarrow r)\rightarrow.p\rightarrow q\rightarrow.p\rightarrow r.$

3) $\{p\rightarrow q, p\} \vdash q$ is a deduction validated by *Aa*;

4) If $\Gamma\cup\{p\} \vdash q$, then $\Gamma \vdash \neg p\vee q$;

5) The following deductions are validated:

$\{p\vee q, \neg p\} \vdash q$

$\{p\vee\neg q, q\} \vdash p$

$\{p\wedge\neg p\} \vdash q$

$\{N\neg p\} \vdash p$ (the endorsement rule: whatever is not wholly false is true)

$\{p, q\} \vdash p\wedge q$

6) The following schemata are not theorematic in *Aa*:

$p\rightarrow q\vee.p\rightarrow Nq$

$p\rightarrow.q\rightarrow p$

$p\rightarrow q\vee.q\rightarrow r$

$p\wedge q\rightarrow r\rightarrow.p\rightarrow.q\rightarrow r$

$p\rightarrow.q\rightarrow.p\wedge q$

$p\wedge Np\rightarrow q$

$p\rightarrow.q\vee Nq$

$\neg p\rightarrow\neg q\rightarrow.q\rightarrow p$

$\neg\neg p\rightarrow p$

$p\rightarrow.q\rightarrow q$

7) The following deductions are not validated by *Aa*:

$p, Np\} \vdash q$

$\{p, Np\vee q\} \vdash q$

$\{p, Np\} \vdash Nq$

$\{\neg p\vee q\} \vdash p\rightarrow q$

$\{p\} \vdash p\wedge q$

$\{p\vee q\} \vdash p$

$\{p\} \vdash q\rightarrow p$

$\{p\rightarrow.q\rightarrow r\} \vdash q\rightarrow.p\rightarrow r$;

8) Defining a (mere) conditional '$\supset$' like this: «$p\supset q$» abbreviates «$\neg p\vee q$», we have:

8i) The set of theorems and deduction rules involving only such wffs as have only essential occurrences of '$\vee$', '$\wedge$' '$\neg$', and '$\supset$' is classical logic;

8ii) p⊃q⊃.Nq⊃Np and the like (p⊃q⊃.p⊃Nq⊃Np, p⊃Nq⊃.q⊃Np etc.) as well as p∧Np⊃q, p⊃.Np⊃q, Np⊃.p⊃q and other versions of «e falso quodlibet» are not theorematic in *Aa*;

9) Defining ‘0’ as abbreviating ‘t∧¬t’, where ‘t’ is a sentential constant, ‘a’ as abbreviating ‘m0’ and ‘1’ as abbreviating ‘N0’ (where ‘0’ is read as ‘the absolutely false,’ ‘1’ as ‘the absolutely true’ and ‘a’ as the infinitesimally true’ or ‘the smallest degree of truth’), we ascertain that the following schemata are theorematic:

p∨0↔p

p∨1↔1

p∧0↔0

p∧1↔p

p↔1∨Np

p↔0∨p

p⊃.a→p

a→p⊃p

p→Na∨Hp

p→Na⊃Np

p→a∧p→.p↔a

10) As for ‘•’, the following principles hold: p•q→p, p•1↔p, p•0↔0, p•q→.p∧q, p•q∨p↔p, p•q↔.q•p, p•q•r↔.p•.q•r, p∨q•r↔.p•r∨p•q r↔.p•r, p⊃.q⊃.p•q; instead, the following schemata are not theorematic:

p•p↔p

p∧q→.p•q

p∨q•p↔p

More interestingly, we have that the following are theorematic (defining «np» as «NmNp»)

n1•p↔np

p→nq∨.q↔mp∨.q→p

np→p

p→mp

n(p•q)↔.np•nq

n(p∧q)↔.np∧nq

n(p∨q)↔.np∨nq

(‘n’ can be read: It’s overtrue that’).

11) In *Aa* we can definitionally introduce infinitely many provably non-equivalent functors — ultimately defined through the primitive symbols of the system — ; in particular we can express infinitely many truth-nuances;

12) *Aa* is a quasi-conservative extension of any finitely many-valued system of logic, i.e. given such system *S* there is a translation which is a monomorphism X carrying symbols of *S* into symbols of *Aa* (wffs being symbols) such that for any formula of *S*, p, p is a theorem of *S* iff H(Xp) is a theorem of *Aa*, wherein 'H' stands for some strong assertive functor or other — the notion of strong assertive functor being defined in a straightforward and intuitively satisfactory way.

We now proceed to the quantificational calculus *Aq*. To *Aa* we add denumerably many variables: x, y, z, u, v, x', ….; plus the (universal) quantifier prefix, '∀'; and we lay down that whenever «p» is a formula, so is «∀up», where 'u' stands for any variable. Then we say that a valuation of *Aq* is a valuation *v* of *Aa* such that, for any wff «p», *v*(∀xp) = g.l.b.{u: there is some x-variant of *v*, *v'*, such that u = *v'*(p)}, where a valuation *v'* is said to be an x-variant of *v* iff for every formula «q» having no occurrence of 'x', v(q) = v'(q).

We are thus enabled to prove such results as (defining «∃xp» as «N∀xNp»):

$\forall xp \to p$

$p \to \exists xp$

$N\exists xNp \leftrightarrow \forall xp$

$\exists xp \to \forall xq \to \forall x(p \to q)$

$\forall xp \wedge \forall xq \leftrightarrow \forall x(p \wedge q)$

$\forall xp \bullet \forall xq \leftrightarrow \forall x(p \bullet q)$

$H\forall xp \leftrightarrow \forall xHp$

$H\exists xp \leftrightarrow \exists xHp$

$\exists xp \vee \exists xq \leftrightarrow \exists x(p \vee q)$

$\neg\forall xp \leftrightarrow \exists x\neg p$

$\neg\exists xq \leftrightarrow \forall x\neg q$

and all standard theorem-schemata involving quantifiers (as well as some schemata peculiar to *Aq*, such as $a \to \forall xp \vee \exists x\neg p$, $\exists xp \to Na \vee \forall xHp$), barring two implicational prenexation principles, which turn out to fail, viz.: $\ulcorner\forall xp \to \forall xq \to \exists x(p \to \forall xq\urcorner)$ and $\ulcorner\forall xp \to \exists xq \to \exists x(\forall xp \to q)\urcorner$ — even though some qualified versions thereof do obtain.

We can prove that, unlike what is the case as regards standard infinite-valued quantificational calculi (such as $G_{\aleph}$ and so on), no stark conservative extension of *Aq* is unsurmountably ω-overinconsistent, i.e. such that every stark conservative extension thereof is such that, were the ω-rule added to its deduction rules, the ensuing system would be deliquescent (where the ω-rule is the rule allowing to draw «∀xp» from a — possibly infinite — set of premises «$p[x/x^1]$», «$p[x/x^2]$», … , *provided* x^1, x^2,… are all the individual constants in the system wherein the rule is applied).

System *Aq* is called a *transitive* logic because its most distinguishing trait is countenancing *transitions*, not just in the sense that it allows for fuzzy situations or fringes between a complete «yes» and complete «no» — fringes that, as befits transitional states, combine or include both the «yes» and the «no», but in a number of degrees which may reach infinity — but also in the sense that for any ordinary situation or state of affairs, p, there are two abutting (immediate) transitions or thresholds thereof: its lower threshold, np, and its upper threshold, mp.

Thus, the inclusion of the nonstandard primitive symbol 'm' among our functors gets justified, in the first place, in virtue of the intuitive idea underlying *transitive logic*: the idea of fringes with a beginning and an end. But it attains a deeper justification by the fact that through it we define a sentential constant 'a' whose existence and properties prevent any ω-overinconsistency. To be sure, there might be alternative ways to the same welcome result — defining 'a' with other primitive symbols, instead of 'm'. Thus, e.g., we could postulate only such standard reals x as $0 \leq x \leq 1$ plus an infinitesimal ∞ (which would take the place of our <0,4>) and its negation N∞ (taking the place of <1,2>, i.e. one nonstandard real infinitely near to 1). Yet, that procedure would give rise to certain anomalies I cannot afford to go into within this paper. As for our nonidempotent (over)conjunction '•', it gets justified, too, by the interesting results we can secure thanks to it, particularly the introducibility of infinitely many defined truth-nuance functors answering to the complexity of natural language more faithfully than standard systems, which are stunted by skimping on the number of primitive symbols. (Conceptual economy is fine; stinginess is always bad, and conceptual stinginess is failing to multiply the primitive symbols when it turns out to be necessary — for conveying interesting truth-nuances.)

We can generalize our formal approach by introducing the notion of a quasi-transitive algebra, q.t.a. for short. A q.t.a. is an algebra <*A*, *T*>, where *T* = <1,H,N,n,∨,•,↔,>, where 1 is a nullary operation, N ,H ,n are unary operations, and∨, ,↔ are binary operations, satisfying the 24 postulate below. First, let's introduce some definitions: 0 eq N1; Sx eq x∧Nx; x∧y eq N(Ny∨Nx) ½ eq 1↔1; mx eq NnNx; ¬x eq HNx; Xx eq x•x; a eq mO; x→y eq x∧y↔x; fx eq ¬(x↔a)∧x; ¥x eq ¬(x→nx)∧Sx.

We also introduce two order relations: x < y means that y = y∨x; x<y means that, while x≤y, x↔y =0.

Let *D* be {x∈*A*: ¬x=0} (the set of dense elements of *A*)

POSTULATES (for any x,y,z,u,v belonging to A):

(01) y∧x∨x=x

(02) x•1=x

(03) XNXx=Nx

(04) ½=N½

(05) x•y≤y∧x

(06) x∧y∧¬(x•y)=0

(07) x↔y ∈ *D* iff x=y

(08) x→y∨(y→nx)∨(x↔my) = ½

(09) ¥x∧fNy∧¥N(x•my) = 0

(10) x•y↔a ≤ x↔a∨(y↔a)

(11) $\neg(x\leftrightarrow 0\vee x) = 0$

(12) $x\leftrightarrow y\leftrightarrow \frac{1}{2}\vee(x\leftrightarrow y\leftrightarrow 0) = \frac{1}{2}$

(13) $\neg(nmx\leftrightarrow nx)\wedge x = 0$

(14) $x\leftrightarrow y \leq x\wedge u\vee z\leftrightarrow(y\vee z\wedge .u\vee z)$

(15) $Hx\wedge Hy = LH(y\wedge x)$

(16) $x\leftrightarrow y \leq z\leftrightarrow y\leftrightarrow(x\leftrightarrow z)$

(17) $z\leftrightarrow y \leq Hx\vee Hz\leftrightarrow H(x\vee y)$

(18) $v\leftrightarrow y < v\bullet(x\wedge u)\bullet z\leftrightarrow(u\bullet z\wedge(x\bullet z)\bullet y)$

(19) $a<\frac{1}{2}$

(20) $x\bullet n1 = nx$

(21) $x\leftrightarrow y\wedge\neg x\wedge y = 0$

(22) $Nx\leftrightarrow y = x\leftrightarrow Ny$

(23) $nx\leftrightarrow mx = x\leftrightarrow a\vee(x\leftrightarrow Na)$

(24) $fSx\wedge Sy \leq \neg(x\wedge y\leftrightarrow(x\bullet y))$

I've elsewhere (in [27] and [28]) investigated algebras closely related to what I here call qq.tt.aa. (the only difference being the present addition of postulate (09); now I'll call algebras fulfilling all the 24 postulates except (09): *transitional algebras*). These algebras evince many mathematically interesting features, especially as regards their relationship with other, more commonly studied, algebras.

The set of alethic elements with the operations we have defined on it is a q.t.a., whose 1-element is <1,3> and whose 0-element is <0,3>.

Now we define a system *Ap* whose vocabulary *V* is {↓,↔,•,a,H} and whose formation rules are: 1) 'a' is a wff; 2) if «p» and «q» are wffs, so are «p↓q», «p↔q», «Hp», «p•q».

We define *T*, the set of theorems of *Ap*, as follows: let A be the carrier of a q.t.a.; then a valuation of *Ap* is a map *v* carrying wffs of *Ap* into A provided for any «p» and «q»: *v*(a) = m0 (where m0 is the least dense element of the q.t.a. in whose carrier is included the range of *v*); *v*(p↓q)=N*v*(p)∧N*v*(q); *v*(p↔q)=*v*(p)↔*v*(q); *v*(p•q) = *v*(p)•*v*(q); *v*(Hp)=H*v*(p). A wff «p» ∈ *T* iff every valuation *v* is such that *v*(p) is a dense element of the q.t.a. in whose carrier the range of *v* is included.

Aa is a nonconservative extension of Ap. *Ap* is more interesting, since some theorems of *Aa* are unmotivated by our philosophical considerations and merely follow from the mathematic peculiarities of the particular q.t.a. of alethic elements. As I pointed out at the beginning of this Section, *Ap* has been axiomatized, thus emerging as a system both sound and complete. Moreover, all the results pointed out above concerning *Aa* can also be proved concerning *Ap.* Therefore, transitive logic is not only philosophically grounded on arguments not easily disposed of but moreover stands out as an interesting formal system.

Ap is a plentiful fuzzy logic (see above, section 3), and hence so is *Aa*, too. We can weaken *Ap (Aa)* into a meagre fuzzy system by removing off 'a' and '↔' (resp. 'm' and '↔') from its primitive symbols and rewording the formation rules accordingly. The ensuing systems are of course less strong than classical logic. Needless to say, when *Ap* is submitted to such impoverishment, the outcome is a system which is not complete in regard to qq.tt.aa. and which lacks most of the interesting properties we've mentioned above among the ones that are provably possessed by *Ap* and Aa.

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